Linear and Angular Speed Worksheet

DJ L-Boogie has a turntable that has two settings, 33 rpm or 45 rpm (revolutions per minute).

1. When the turntable is set at 33 revolutions per minute (rpm), what is its angular speed in radians per minute?

$$\frac{\theta}{t} = \frac{33 \cdot 2\pi}{1 \, min} = 66\pi \text{ rad/min}$$

2. When the turntable is set at 45 rpm, what is its angular speed in radians per minute?

$$\frac{\theta}{t} = \frac{45 \cdot 2\pi}{1 \, min} = 90\pi \text{ rad/min}$$

While a record is spinning, a ladybug lands on the turntable 10 inches from the center.

3. What is the linear speed (in inches per minute) of the ladybug when the turntable is set to:

    a) 33 rpm   **b)** 45 rpm

$$10 \cdot 66\pi \qquad\qquad\qquad 10 \cdot 90\pi$$

$$\boxed{2073.45 \text{ inches/min}} \qquad \boxed{2827.43 \text{ inches/min}}$$

4. The ladybug crawls towards the middle and is now 3 inches from the center, what is the linear speed (in inches per minute) of the ladybug when the turntable is set to:

    a) 33 rpm   **b)** 45 rpm

$$3 \cdot 66\pi \qquad\qquad\qquad 3 \cdot 90\pi$$

$$\boxed{622.04 \text{ inches/min}} \qquad \boxed{848.23 \text{ inches/min}}$$

5. Convert the speed of the bug in problem 3 at 33 rpm to miles per hour.

$$\frac{2073.45 \text{ inches}}{1 \, min} \cdot \frac{60 \, min}{1 \, hr} \cdot \frac{1 \, ft}{12 \, in} \cdot \frac{1 \, mile}{5280 \, ft} = \frac{124407}{63360} = \boxed{1.96 \text{ mph}}$$

6. Convert the speed of the bug in problem 4 at 45 rpm to miles per hour.

$$\frac{848.23 \text{ inches}}{1 \, min} \cdot \frac{60 \, min}{1 \, hr} \cdot \frac{1 \, foot}{12 \, inches} \cdot \frac{1 \, mile}{5280 \, ft} = \frac{50893.8}{63360} = \boxed{.80 \text{ mph}}$$

7. A 16mm diameter shaft rotates at 1,500 rps (revolutions per second). Find the speed of a particle on its surface (to the nearest meter per second).

Linear Speed $r = 8mm$

$$\frac{r\theta}{t} = \frac{8 \cdot 1500 \, (2\pi) \, mm}{1 \, sec} \cdot \frac{1 \, meter}{1000 \, mm} = 75.4 \text{ meters/sec}$$

8. An earth satellite travels in a circular orbit at 20,000 mph. If the radius of the orbit is 4,300 mi, what angular velocity (in radians per hour, to one decimal place) is generated?

$$\frac{r\theta}{t} = \frac{20\,000\,m}{1\,hr}$$

$$4300\,\theta = 20000\,miles/hr$$

$$\theta = 4.7 \quad radians/hr$$

9. The earth revolves about the sun in an orbit that is approximately circular with a radius of 9.3×10^7 mi. The radius of orbit sweeps out an angle with what exact angular velocity (in radians per hour)? How fast (to the nearest hundred miles per hour) is the earth traveling around its orbit. Hint: it takes the earth 365 days to complete its orbit.

$$Angular = \frac{\theta}{t} = \frac{2\pi}{365\,days} \cdot \frac{1\,day}{24\,hrs} = \frac{2\pi}{8760} = \boxed{\frac{\pi}{4380}\ radians/hr}$$

$$Linear = \frac{9.3 \times 10^7\,\pi\,miles}{4380\ hour} = \boxed{66705.05\ mph}$$

10. The second hand on Mr. Incredible's watch is .25 inches long. How fast is the tip of the second hand moving? Give your answer in inches per second.

$$Linear\ speed \quad \frac{r\theta}{t} = \frac{.25\,(1)\,2\pi}{60\,sec} = \boxed{.026\ /sec}$$

11. A neighborhood carnival has a Ferris wheel whose radius is 30 feet. You measure the time it takes for one revolution to be 70 seconds. What is the linear speed (in feet per second) of this Ferris wheel? What is the angular speed in radians per second?

$$Angular$$
$$\frac{2\pi}{70} = \boxed{\frac{\pi}{35}\ radians/sec}$$

$$Linear$$
$$\frac{30\,\pi}{35} = \boxed{2.6928\ feet/sec}$$

12. A spin balancer rotates the wheel of a car at 480 revolutions per minute. If the diameter of the wheel is 26 inches, what road speed is being tested? Express your answer in miles per hour. At how many revolutions per minute should the balancer be set to test a road speed of 80 miles per hour?

See Attached

13. A Ford Expedition comes standard with tires that have a diameter of 25 inches. If the owner decided to upgrade to tires with a diameter of 30 inches without having the onboard computer updated, how fast will the Expedition actually be traveling when the speedometer reads 75 mph?

See Attached

(12) Linear (mph)

$$\frac{r\theta}{t} = \frac{13(2\pi)(480) \text{ in}}{1 \text{ min}} \cdot \frac{60 \text{ min}}{1 \text{ hr}} \cdot \frac{1 \text{ ft}}{12 \text{ in}} \cdot \frac{1 \text{ mile}}{5280 \text{ ft}}$$

$$\boxed{= 37.12 \text{ mph}}$$

$$\frac{80 \text{ miles}}{1 \text{ hr}} = \frac{13(2\pi) X \text{ in}}{1 \text{ min}}$$

Given linear speed find # of rotations

$$\frac{80 \text{ miles}}{1 \text{ hr}} \cdot \frac{1 \text{ hr}}{60 \text{ min}} \cdot \frac{5280 \text{ ft}}{1 \text{ mile}} \cdot \frac{12 \text{ in}}{1 \text{ ft}} = \frac{26\pi X \text{ in}}{1 \text{ min}}$$

$$\frac{84480 \text{ in}}{1 \text{ min}} = \frac{26\pi X \text{ in}}{1 \text{ min}}$$

$$\frac{84480}{26\pi} = \frac{26\pi X}{26\pi}$$

$$\boxed{1034.26 \text{ rotations/min}}$$

(13)   25 in diam.   30 in diam

    75 mph → linear speed   15(2π)(60504.343) in
    find # of rotations   ───────────────────────
                                            1 hr

• first convert linear   1815130.29π in   1ft   1mile
   speed to inches   ─────────────── · ──── · ─────
                                      1 hr   12in   5280ft

  75 miles   5280 ft   12in   89.99
  ──────── · ───────── · ────
    1 hr   1mile   1ft   ┌──────────────┐
                                     │ ≈ 90 mph   │
  4752000 in/hr   └──────────────┘

          rθ
  Linear = ──
           t

  4752000 in   12.5(2π) X in
  ────────── = ────────────────
    1 hr   1 hr

  4752000 in   25π X in
  ────────── = ──────────
    25π   25π

  ┌─────────────────────────────────────┐
  │ X = 60504.343 rotations/hr   │
  └─────────────────────────────────────┘

Precalculus 4.1 Review

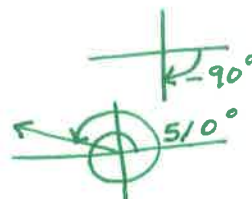Draw an angle with the given measure in standard position.
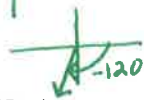
11. 75°
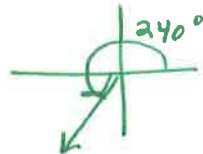
12. 160°

13. −90°

14. −120°

15. 295°

16. 510°

17. **GYMNASTICS** A gymnast on the uneven bars swings to make a 240° angle of rotation.

18. **FOOD** The lid on a jar of pasta sauce is turned 420° before it comes off.

Find an angle with a positive measure and an angle with a negative measure that are coterminal with each angle.

19. 50° $410°, -310°$

20. 95° $455°, -265°$

21. 205° $565°, -155°$

22. 350° $710°, -10°$

23. −80° $280°, -440°$

24. −195° $165°, -555°$

Rewrite each degree measure in radians and each radian measure in degrees.

25. 330° $\frac{11\pi}{6}$
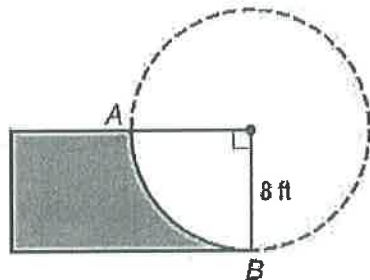
26. $\frac{5\pi}{6}$ $150°$

27. $-\frac{\pi}{3}$ $-60°$

28. −50° $-\frac{5\pi}{18}$

29. 190° $\frac{19\pi}{18}$

30. $-\frac{7\pi}{3}$ $-420°$

31. **SKATEBOARDING** The skateboard ramp at the right is called a *quarter pipe*. The curved surface is determined by the radius of a circle. Find the length of the curved part of the ramp. $12.6 \, ft$

32. **RIVERBOATS** The paddlewheel of a riverboat has a diameter of 24 feet. Find the arc length of the circle made when the paddlewheel rotates 300°.
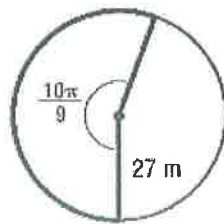$62.8 \, ft$

8 ft

Find the length of each arc. Round to the nearest tenth.

33.

5 cm $\frac{3\pi}{7}$ $6.7 \, cm$

34.
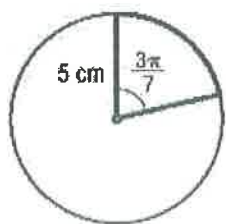
$\frac{10\pi}{9}$ 27 m $94.2m$

35. **CLOCKS** How long does it take for the minute hand on a clock to pass through 2.5π radians? $1 hr \, 15 min$

136. **SUNDIALS** Refer to the beginning of the lesson. A shadow moves around a sundial 15° every hour.

  a. After how many hours is the angle of rotation of the shadow $\frac{8\pi}{5}$ radians? $19.2 \, hours$

  b. What is the angle of rotation in radians after 5 hours? $\frac{5\pi}{12}$

  c. A sundial has a radius of 8 inches. What is the arc formed by a shadow after 14 hours? Round to the nearest tenth. $29.3 \, in$